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Mathematical modelling and investigation of the grain drying process in bunker units with radial supply of drying agent

Abstract. One of the key tasks of modern grain processing equipment is to increase the efficiency of drying in bunker installations with a ring-cylindrical grain bed. The aim of this study was to investigate the unsteady processes of heat and mass transfer in the annular layer of grain in a bin-type dryer with a radial supply of drying agent to assess the uniformity of temperature and moisture distribution and determine the effect of drying parameters on dehydration efficiency. To improve the energy efficiency and quality of drying, a two-level mathematical model was proposed to study the processes of internal heat and mass transfer and external heat and mass transfer. At the first level, the drying kinetics of a single grain in the form of a sphere in contact with a drying agent of constant temperature and humidity was modelled. The second level described the process of filtration drying of a dense ring-cylindrical layer of grain with a radial supply of the

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drying agent. The thermophysical coefficients of the models were identified based on the results of field experiments. The obtained analytical and numerical solutions of the presented mathematical models made it possible to determine the rational design parameters and calculate energy-efficient operating modes of bunker-type grain dryers depending on the initial grain parameters. It was substantiated that the highest efficiency is achieved when using a periodic circulation mode, which ensures uniform heating and dehydration of the grain mass along the radial coordinate of the bed. The dependences of the influence of temperature, moisture content and air filtration rate on the exposure of grain drying in the bed were determined, which allows for flexible control of the drying process depending on the operating conditions. The study results confirmed that the use of a batch-circulating drying mode allows achieving high uniformity of the final grain moisture content, increasing the energy efficiency of the process and significantly reducing the unevenness of grain heating compared to traditional continuous modes of drying grain in a thick radial layer. The results can be used both for the modernisation of existing drying plants and for the design of new energy-saving grain dryers

Keywords: mathematical modelling; heat and mass transfer; bunker dryer; cyclic-periodic mode; ring-cylindrical layer

INTRODUCTION

Grain drying is a mandatory post-harvest process that serves as a crucial step in preparing crops for long-term storage and subsequent processing. Freshly harvested grain continues to undergo post-harvest ripening processes; therefore, drying not only reduces moisture content but also accelerates maturation, inhibits the development of pathogenic microflora, and homogenises the grain mass in terms of ripeness. Drying also significantly enhances the technological efficiency of subsequent grain processing in mills and groat factories.

The primary method for grain drying in agricultural enterprises is thermal drying with convective heat supply. Despite its numerous advantages, this method has significant drawbacks, including the high energy consumption of the process and the low efficiency of dryers due to substantial heat losses with the exhaust drying agent. Consequently, optimising the parameters of the convective drying process is critically important by H.S. El-Mesery *et al.* (2024). Most grain drying installations utilise straight-through shaft designs with gas distribution boxes, commonly known as shaft dryers, with capacities ranging from 24 to 50 tons per hour. These dryers are characterised by their high metal consumption and uneven grain heating as shown in the study by P. Demissie *et al.* (2019). This unevenness arises from the large contact area between the distribution boxes and the grain, leading to localised overheating. Such localised overheating significantly contributes to drying non-uniformity. Non-uniform drying poses significant microbiological risks and ultimately results in a reduction in the quality of the final dried product.

In high-temperature convective drying, the distribution of the air flow and the heat carrier supply modes significantly impact drying efficiency. According to research by W. Wang *et al.* (2020), implementing an effective heat carrier distribution within the grain layer can reduce localised overheating and increase drying efficiency. The high throughput of shaft dryers complicates the processing of small grain batches, as highlighted in a review by K.A. Jimoh *et al.* (2023), which emphasised the low flexibility of such installations for decentralised processing. For micro, small, and medium-sized enterprises involved in grain production, the application of column and batch dryers presents an effective solution. Studies by A. Sokolenko *et al.* (2020) demonstrated that these dryers offer lower metal consumption and cost while providing sufficient capacity. Similar conclusions were drawn by A.I. Onderová *et al.* (2019), who investigated the drying processes of biomaterials in vacuum dryers and noted their adaptability to various grain types and batch volumes. The use of column dryers in conjunction with heat generators powered by solid alternative fuels significantly reduces drying costs. However, this approach necessitates a separate bunker for grain cooling. This requirement was analysed by Z.Y. Xu *et al.* (2019), who explored the prospects of low-temperature heat recovery and its potential application in grain drying infrastructure.

Improving the technological and energy efficiency of grain dryers is intrinsically linked to the intensification of heat and mass transfer processes and the utilisation of new heat supply schemes for grain material. The fundamentals, methods, and means of drying intensification were identified by I. Palamarchuk *et al.* (2023). Among the effective methods are: vibro-loosening of the grain layer, non-contact infrared (IR) and microwave energy supply, and ozonation of the drying agent. These methods substantially accelerate the dehydration process, thereby increasing dryer productivity. While microwave drying offers some advantages, it also presents limitations, including uneven heating, limited penetration into thick materials, and the risk of surface damage due to excessively rapid moisture evaporation. Therefore, researchers N. Ahmad *et al.* (2021) were studying the combined application of microwave drying with other technologies to integrate the benefits of each method, enhance drying efficiency, and improve grain quality. Combining microwave and infrared drying enhances process efficiency and optimises energy utilisation. Microwave heating penetrates deep into the material, heating it through the oscillation of water molecules, while infrared radiation primarily heats the surface, rapidly increasing its temperature. This combined approach was investigated by P. Mukwevho & M.N. Emmambux (2022). Infrared (IR) drying technology is a promising method for moisture removal due to its high energy conversion efficiency and direct transfer of thermal energy to the material's surface. V.R.G. Nascimento *et al.* (2019) noted that unlike traditional hot-air drying, IR drying minimises heat losses to the environment, ensures a rapid increase in product temperature, and allows operation at lower temperatures. This contributes to a reduced process duration and decreased energy consumption.

However, the limited penetration depth of IR radiation reduces its effectiveness when processing thick-layer or low-moisture materials. Therefore, N.D.S. Timm *et al.* (2020) combined this technology with other drying methods to enhance the overall process efficiency. P. Yu *et al*. (2025) determined that one common challenge with the aforementioned methods for increasing dryer productivity is that they often lead to higher capital or operational costs due to increased power requirements, equipment complexity, and energy consumption. An alternative approach is to optimise dryer operating modes and technological schemes to ensure stable productivity with minimal energy consumption.

This optimisation necessitates accurate mathematical models that describe heat and mass transfer in specific drying configurations. Research by O. Tsurkan *et al.* (2022) confirmed the feasibility of using combined analytical and numerical modelling methods for convective drying to predict temperature and moisture fields. However, aspects such as the distribution of temperature and moisture along the radius of thick-layer annular dryers with radial air flow remain insufficiently studied.

The aim of this study was to investigate unsteady heat and mass transfer processes in the annular grain layer of a bunker-type dryer with radial supply of the drying agent to assess temperature and moisture distribution uniformity and evaluate the impact of drying parameters on dehydration efficiency.

Research objectives:

- to develop a mathematical model of unsteady heat and mass transfer in a ring-cylindrical layer of grain mass, taking into account changes in the radial coordinate of temperature, moisture content, and filtration rate of the drying agent;
- to identify thermo physical coefficients (effective thermal conductivity, effective diffusion), similarity criteria (heat and mass transfer Bio) from classical analytical solutions of heat and mass transfer equations for a spherical body to calculate the heating and drying coefficients of the external heat and mass transfer model.

The scientific novelty of the study was that a mathematical model of unsteady heat and mass transfer in the grain layer of a bunker dryer was developed, taking into account changes in the radial flow rate of the drying agent and the empirical dependence of grain equilibrium moisture content on drying parameters. The model allowed identifying the distribution patterns of temperature and humidity along the bed height and radius, as well as the risk zones of overdrying or underdrying. A new approach to reducing uneven drying by means of cyclic grain mixing was substantiated, suitable for implementation in single or serially connected bunker-type dryers.

MATERIALS AND METHODS

To determine the rational modes of drying grain (seed) material and optimise the technological and structural parameters of grain dryers, it was necessary to study the kinetics of dehydration and heating of both single grains and the entire dense layer of grain in its interaction with the drying agent. The drying kinetics of individual grains was determined by interrelated processes of internal heat and mass transfer, and the drying of the grain layer was determined by the hydrodynamic regime of drying agent filtration in the intergrain space of the moving material. At the same time, the parameters of the drying agent changed both in time and in the coordinates of the drying volume of the grain dryer. Therefore, the research was based on an analytical and experimental method based on a two-level mathematical model of internal heat and mass transfer and external heat and moisture exchange of grain with the drying agent.

At the first level (micro level), a mathematical description of the drying kinetics (changes in moisture content and temperature) of a single spherical grain in the form of a ball when it interacts with a drying agent with time-variable parameters was formulated. The thermophysical coefficients of the mathematical model were identified from the experimental data. At the second level (macro kinetics), a two-stream mathematical model of heat and mass transfer of the drying agent, variable parameters, and a gravitationally moving ring-cylindrical grain layer during radial filtration of the drying agent was formulated.

The interconnected transfer of heat and mass (in the form of liquid and vapor) inside a wet material is described, as is well known, by the system of differential equations by I. Palamarchuk *et al.* (2023), which were written in the form:

$$\begin{cases} \frac{\partial \theta}{\partial \tau} = a_T \nabla^2 \theta + \frac{\varepsilon r_v}{c} \frac{\partial U}{\partial \tau} \\ \frac{\partial U}{\partial \tau} = a_m \nabla^2 U + a_m \delta_T \nabla^2 \theta + \varepsilon \frac{\partial U}{\partial \tau}, \end{cases} \tag{1}$$

where θ, U – temperature and moisture content of the material; a_T, a_m, δ_T – coefficients of thermal conductivity, moisture conductivity, and thermo gradient coefficient; ε – the phase transformation coefficient; c – specific heat capacity of the material; r_v – specific heat of vaporisation; $\nabla\theta$, ∇U – gradients of the temperature and moisture content field in the body.

The system of equations (1) can be solved numerically with the defined coefficients a_m, a_T, ε, δ_T. However, existing methods for determining the transfer coefficients involve separate experiments for each coefficient, which leads to rather complicated empirical dependencies that complicate calculations and reduce their reliability. The problem arose of determining the coefficients of the equations from experimental information using known analytical solutions of the equations of thermal conductivity and mass transfer separately. To do this, the equations of system (1) were simplified.

The second term on the right-hand side of the first equation determines the amount of heat for moisture evaporation. Using the Rebinder criterion $Rb = \frac{c}{r}\cdot\frac{\partial\theta}{\partial U}$, was replaced $r\cdot\left(-\frac{\partial U}{\partial\tau}\right) = \frac{c}{Rb}\cdot\frac{\partial\theta}{\partial\tau}$. The coefficient of thermal and moisture conductivity is set to zero, given the practical absence of thermogradient moisture transfer in the grain under the adopted drying regimes.

It was supposed that the grain shape is a sphere with an equivalent radius:

$$R = \frac{1}{2}\sqrt{\frac{6V_g}{\pi}}, \tag{2}$$

where V_g – volume of the grain.

Taking into account the adopted simplifications, the equations of heat and mass transfer in a single grain can be written in the form:

$$\frac{\partial\theta(r,\tau)}{\partial\tau} = a_{ef}\cdot\left(\frac{\partial^2\theta(r,\tau)}{\partial^2 r}+\frac{2}{r}\frac{\partial\theta(r,\tau)}{\partial r}\right); \tag{3}$$

$$\frac{\partial U(r,\tau)}{\partial\tau} = a_{mef}\cdot\left(\frac{\partial^2 U(r,\tau)}{\partial^2 r}+\frac{2}{r}\frac{\partial U(r,\tau)}{\partial r}\right), \tag{4}$$

where $a_{ef} = \frac{a_T}{1-\frac{\varepsilon}{Rb}}$ – coefficient of effective thermal conductivity; $a_{mef} = \frac{a_m}{1-\varepsilon}$ – coefficient of effective moisture conductivity; $\varepsilon = \frac{dU_{ev}}{dU}$; r – coordinate of the measurement point.

Interphase heat and moisture exchange (grain – drying agent) occurs by convective-diffusion, therefore, boundary conditions of the third kind were used to solve equations (3) and (4):

$$-\frac{\partial\theta(R,\tau)}{\partial r}+\frac{\alpha}{\lambda}\cdot(t_c-\theta(R,\tau)) = 0;\ \frac{\partial\theta(R,\tau)}{\partial r} = 0, \tag{5}$$

$$\frac{\partial U(R,\tau)}{\partial R}-\frac{\beta}{a_m}\cdot\left(U(R,\tau)-U_{riv}(t,\varphi)\right) = 0;\ \frac{\partial U(R,\tau)}{\partial R} = 0. \tag{6}$$

Initial conditions:

$$\theta(\bar{R},\bar{\tau}) = \bar{\theta}_0;\ U(\bar{R},\bar{\tau}) = \bar{U}_0. \tag{7}$$

The solutions of equations (3), (4) under boundary conditions (5-7) are known (Kaletnik *et al.*, 2024) and for changes in time of the average (over the radius) values of the sought parameters $\theta(\tau)$ and $U(\tau)$ take the form:

$$\bar{\theta}(\tau) = t_c-(t_c-\theta_0)\cdot\sum_{n=1}^{\infty}\frac{6\cdot Bi^2}{\mu_n^2\cdot[\mu_n^2+Bi^2-Bi]}\cdot e^{-\frac{a_{ef}}{R^2}\cdot\tau\cdot\mu_n^2}, \tag{8}$$

$$\bar{U}(\tau) = U_{riv}+(U_0-U_{riv})\cdot\sum_{n=1}^{\infty}\frac{6\cdot Bi_m^2}{\mu_n^2\cdot[\mu_n^2+Bi_m^2-Bi_m]}\cdot e^{-\frac{a_{mef}}{R^2}\cdot\tau\cdot\mu_n^2}, \tag{9}$$

$$tg\mu = -\frac{\mu}{Bi-1}, \tag{10}$$

where μ_n – roots of characteristic equation (10); $Bi = \frac{\alpha\cdot R}{\lambda}$; $Bi_m = \frac{\beta\cdot R}{a_{mef}}$ – heat and mass transfer criterion "Bio"; α – heat transfer coefficients (W/(m²·K)); β – moisture transfer coefficient (m/s); λ – thermal conductivity coefficient, W/(m·K); a_{mef} – coefficient of effective moisture conductivity (m²/s); a_{ef} – coefficient of effective thermal conductivity (m²/s); t_c – temperature of drying agent (°C); U_{riv} (t, φ) – equilibrium moisture content (kg/kg); φ – relative humidity of drying agent (%).

To reduce the influence of random experimental errors on the result of calculating the coefficients of equations (8), (9), the obtained experimental kinetics data were approximated by dependencies in the form of equations:

$$\theta(\tau) = t_2-(t_2-\theta_1)\ e^{-k_1\tau}; \tag{11}$$

$$U(\tau) = U_2+(U_1-U_2)\ e^{-k_2\tau}, \tag{12}$$

where θ_1, U_1, t_2, U_2 – initial and final values of the parameters; k_1, k_2 – heating and dehumidification coefficients, respectively. For calculations and numerical modelling, the mathematical software Mathcad was used, enabling the solution of transcendental equations and implementation of the Levenberg-Marquardt algorithm via the genfit function. The Statistica 10 package was employed for approximation and construction of empirical dependencies, allowing for highly accurate polynomial models with strong correlation coefficients.

RESULTS AND DISCUSSION

To search for unknown coefficients in equations (8), (9) using the inverse problem method, numerous solutions of the transcendental equation (10), obtained in the mathematical environment Mathcad, are approximated by empirical dependencies. Figure 1 shows a screen fragment of the numerical solution of equation (10).

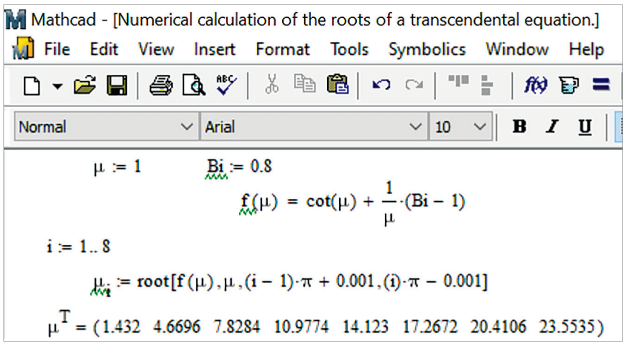

Figure 1. Numerical determination of the roots of the transcendental equation (10) in the mathematical package Mathcad

Source: compiled by the authors

For μ_1, after numerical experiments with various approximating mathematical structures in the Statistica 10 package, an empirical dependence with the highest level of multiple correlation (R = 0.99981) of the form:

$$\mu_1 = b_0+b_1\ \mathrm{Bi}+b_2\ \mathrm{Bi}^{b_3}, \tag{13}$$

where the coefficients: b_0 = -0.090821; b_1 = -0.27041; b_2 = 1.924477; b_3 = 0.461608; Bi = 0.02 ÷ 10.5. For μ_2 - μ_6 the approximating dependences on the Bio criterion are defined as a polynomial of the second degree:

$$\mu = b_0+b_1\ \mathrm{Bi}+b_2\ \mathrm{Bi}^2. \tag{14}$$

The values of the dependence coefficients (14) were determined in the Statistica 10 package and are given in Table 1.

Table 1. Values of dependence coefficients (13)

μ	b_0	b_1	b_2	R (multiple correlation coefficient)
$\mu 2$	4.49427	0.228132	-0.0108	0.9998
$\mu 3$	7.72303	0.136715	-0.00428	0.9999
$\mu 4$	10.90255	0.09611	-0.00206	0.9999
$\mu 5$	14.06520	0.07373	-0.00112	0.9999
$\mu 6$	17.22012	0.05973	-0.00067	0.9999

Source: compiled by the authors

Substitute the determined dependencies μ_1 - μ_6 into dependency (8) and (9) and, using the built-in genfit function in the Mathcad mathematical package implementing the Levenberg-Marquardt algorithm. Determine the values of Bi, Bi_m and a_{ef}, a_{mef} at which dependencies (8) and (9) will most accurately describe the sets of given points determined from (11) and (12). Take the initial a priori approximate values of Bi, Bi_m and a_{ef}, a_{mef} for the given parameters of the heating and drying process in the elementary layer of wheat grain from reference books and preliminary studies (Kaletnik *et al.*, 2024). A screen fragment of calculations of *a* and *Bi* by formulas (8), (13)-(14) in the Mathcad package is illustrated in Figure 2.

Figure 3 shows graphs of changes in moisture content and grain temperature in a layer 0.02 m thick during air filtration with a temperature of t_c = 60°C and a velocity at the entrance to the layer of 0.22 m/s. The graphs show points obtained from empirical dependencies (11) and (12) and a curve determined by the genfit function in MathCad according to equations (8) and (9).

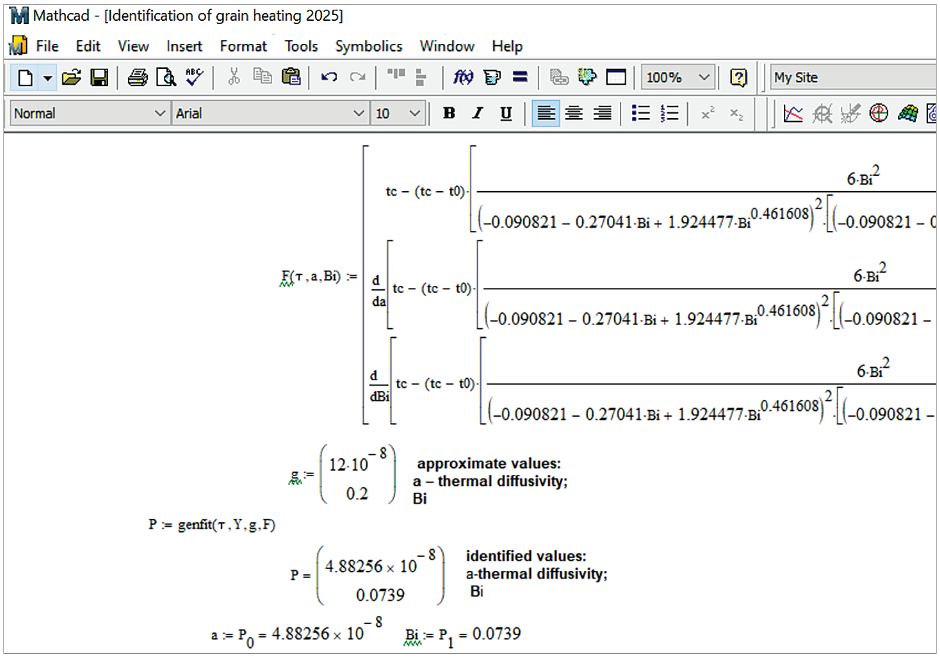

Figure 2. Screen fragment of the implementation of the genfit function in the Mathcad mathematical package for determining the thermal diffusivity coefficient a and the criterion *Bi*

Source: compiled by the authors

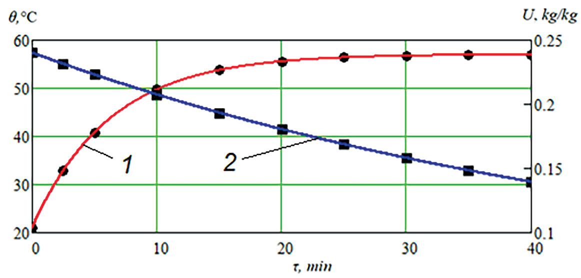

Figure 3. Kinetics of grain temperature 1 and grain moisture content 2

Note: temperature of drying agent t_c = 60°C; velocity at the entrance to the layer υ_{r_0} = 0.22 m/s

Source: compiled by the authors

For further analytical studies, simplify equations (8) and (9). As is known, in the stage of regular mode, the functions that describe the drying and heating processes can be described by one (first) term of the series and the dependence (8) and (9) can be presented in a simplified form:

$$\bar{\theta}(\tau) - \bar{\theta}_0 = B_1 \cdot (t_c - t_0) \cdot e^{-\frac{a_{ef}}{R^2}\tau\mu_1^2}; \qquad (15)$$

$$\bar{U}(\tau) - \bar{U}_p = B_2 \cdot (U_0 - U_p) \cdot e^{-\frac{a_{mef}}{R^2}\tau\mu_1^2}, \qquad (16)$$

where $B_1 = \frac{6\cdot Bi^2}{\mu_1^2\cdot[\mu_1^2+Bi^2-Bi]}$; $B_2 = \frac{6\cdot Bi_m^2}{\mu_1^2\cdot[\mu_1^2+Bi_m^2-Bi_m]}$;
$\mu_1^2 = \left(\frac{1}{\pi}+\frac{1}{3\cdot Bi_m}\right)^{-1} \approx \left(\frac{1}{3}+\frac{1}{Bi}\right)^{-1}$.

The rate of change of temperature and moisture content of grain in time was determined by differentiating equations (15) and (16) with respect to time. After differentiating equations (15) and (16) and comparing the obtained derivatives with (15) and (16), B_1 and B_2 were eliminated, and the following was obtained:

$$\frac{d\bar{\theta}(\tau)}{d\tau} = k_t \cdot (t_c - \theta(\tau)); \qquad (17)$$

$$\frac{d\bar{U}(\tau)}{d\tau} = -k_c \cdot \left(\bar{U}(\tau) - U_{riv}(t, \varphi)\right), \qquad (18)$$

where $k_t = \frac{a_{ef}}{R^2}\left(\frac{1}{\pi}+\frac{1}{3\cdot Bi_m}\right)^{-1}$; $k_c = \frac{a_{mef}}{R^2}\left(\frac{1}{\pi}+\frac{1}{3\cdot Bi_m}\right)^{-1}$.

The equilibrium moisture content of grain is usually determined as a function of temperature t and relative humidity φ of the drying agent and can be calculated using the empirical formula obtained from research:

$$U_{riv}(t, \varphi) = b_0 + b_1 \cdot t + b_2 \cdot \varphi, \qquad (19)$$

where $b_0 = 0.02343$; $b_1 = 0.114 \cdot 10^{-3}$; $b_2 = 0.002067$; t – temperature of the drying agent (°C); φ – relative humidity of the drying agent (%). But a more informative parameter of air is the moisture content d, therefore formula (18) is approximated as dependence:

$$U_{riv}(t, d) = b_0 \cdot t^{b_1} \cdot d^{b_2}, \qquad (20)$$

where $b_0 = 0.725$; $b_1 = -1.2133$; $b_2 = 0.8826$; d – absolute humidity of the drying agent (g/m³). Equations (15) and (16) are obtained under the condition that the temperature of the drying agent is constant over time. However, when air is filtered through a dense layer of grain material, its temperature changes both in time and along the coordinate r.

The calculation diagram of the grain material drying process in a bunker dryer with radial supply of the drying agent is shown in Figure 4.

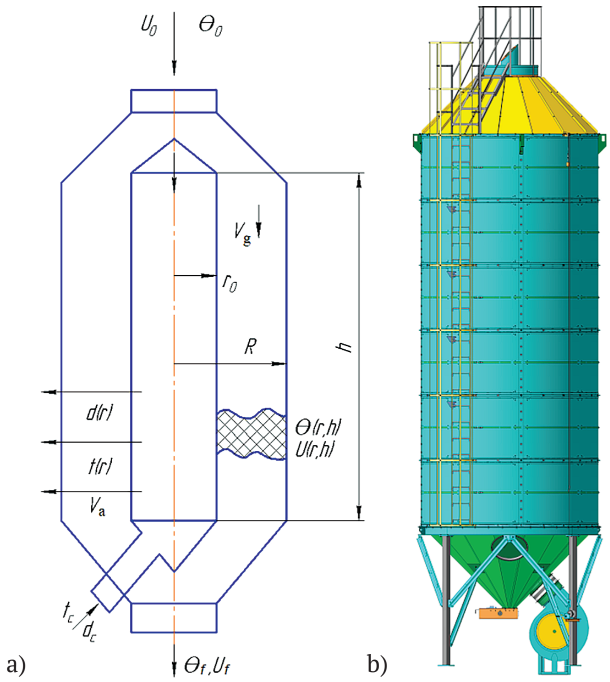

Figure 4. Calculation diagram of the grain material drying process in a bunker dryer with radial supply of drying agent (a) and general view of the bunker type dryer (b)

Note: U_0, is the initial moisture content of grain, kg/kg; U_f is the final average moisture content of grain, kg/kg; θ_0 is the initial temperature of grain, °C; θ_f is the final average temperature of grain, °C; r_0 is the radius of the inner cylinder of the bunker, m; R is the radius of the outer cylinder of the bunker, m; r is a variable radial coordinate, m; h is a variable vertical coordinate, m; t_c is the temperature of the drying agent at the inlet to the grain layer, °C; d_c is the moisture content of the drying agent at the inlet to the grain layer, g/m³; $t(r)$ is the variable temperature of the drying agent at the radial coefficient, °C; $d(r)$ is the variable moisture content of the drying agent at the radial coefficient, g/m³; $V_a(r)$ is the filtration rate of the drying agent in the grain layer, at the radial coordinate, m/s; $V_g(h)$ is the speed of grain movement, by vertical coordinate, m/min; $\theta(r, h)$ is the temperature of grain by radial and vertical coordinates, °C; $U(r, h)$ is the moisture content of grain by radial and vertical coordinates, kg/kg
Source: compiled by the authors

To formulate a mathematical model of unsteady heat and mass transfer in the process of grain drying in a moving annular-cylindrical layer (macro level model), will expand the full differentials in equations (17) and (18) were expanded: $d\theta = \frac{\partial\theta}{\partial\tau}d\tau + \frac{\partial U}{\partial h}dh$; $dU = \frac{\partial U}{\partial\tau}d\tau + \frac{\partial U}{\partial h}dh$. Considering that $\frac{dh}{d\tau} = \upsilon_{gr}$ – velocity of grain material and using (19) the following was obtained:

$$\frac{\partial\theta}{\partial\tau} + \upsilon_{gr}\frac{\partial\theta}{\partial h} = k_t \cdot (t_c - \theta); \quad (21)$$

$$\frac{\partial U}{\partial\tau} + \upsilon_{gr}\frac{\partial U}{\partial h} = -k_c \cdot (U - b_0 \cdot t^{b_1} \cdot d^{b_2}). \quad (22)$$

The heat balance equation for the drying agent can be written as:

$$-m_p \cdot c_p \cdot d\bar{d} = m_{gr} \cdot c_{gr} \cdot d\bar{\theta} - m_0 \cdot r \cdot d\bar{U}. \quad (23)$$

Considering the complete differentials $d\bar{d}, d\bar{\theta}, d\bar{U}$ and taking into account the obvious signs: $m_{gr} = V_{sp} \cdot \rho_{gr}$; $m_0 = V_{sp} \cdot \rho_0$; $m_p = V_p \cdot \rho_p$; $\varepsilon = V_p/V_{sp}$, equation (23) was rewritten as:

$$\varepsilon \cdot \rho_p \cdot c_p \cdot \left(\frac{\partial t}{\partial\tau} + \upsilon_r\frac{\partial t}{\partial r}\right) = \rho_{gr} \cdot c_{gr} \cdot \left(\frac{\partial\theta}{\partial\tau} + \upsilon_{gr}\frac{\partial\theta}{\partial h}\right) - \rho_0 \cdot r_0 \cdot \left(\frac{\partial U}{\partial\tau} + \upsilon_{gr}\frac{\partial U}{\partial h}\right). \quad (24)$$

The equation for the material balance sheet was similar:

$$\varepsilon \cdot \rho_p \cdot \left(\frac{\partial d}{\partial\tau} + \upsilon_r\frac{\partial d}{\partial r}\right) = \rho_0 \cdot \left(\frac{\partial U}{\partial\tau} + \upsilon_{gr}\frac{\partial U}{\partial h}\right). \quad (25)$$

Thus, the dynamic drying regime of grain material was determined by solving the system of equations (21, 22, 24, 25) under the following boundary and initial conditions:

$$U(0, h) = U_0;\ \theta(0, h) = \theta_0;\ t(\tau, 0) = t_1;\ d(\tau, 0) = d_1. \quad (26)$$

In equations (24)-(25), the value $V_r = V(r)$ is the actual filtration speed of the drying object through the layer. The overall, calculated for the entire cross-section of the unfilled volume υ_r is related to the actual speed of the drying agent by the ratio $\upsilon_{c,a} = \frac{\upsilon_r(r)}{\varepsilon}$, and the overall speed varies along the coordinate in the direction of movement:

$$\upsilon_r = \frac{L_{c.a}}{2 \cdot \pi \cdot H \cdot r}, \quad (27)$$

where $L_{c.a}$ – volumetric flow rate of drying agent; H – height of the working zone of the grain dryer. In the stationary mode of operation of the drying unit, the process parameters at each point of the drying volume will have values that do not vary in time; therefore, time derivatives can be excluded from the differential equations.

The stationary operating mode of the grain drying process in a gravitationally moving annular-cylindrical layer describes the following level system:

$$\begin{cases} \upsilon(r)\frac{\partial t}{\partial r} = \frac{\rho_{gr} \cdot c_{gr}}{\varepsilon \cdot \rho_p \cdot c_p}\upsilon_{gr}\frac{\partial\theta}{\partial h} - \frac{\rho_0 \cdot r_0}{\varepsilon \cdot \rho_p \cdot c_p}\upsilon_{gr}\frac{\partial U}{\partial h} \\ \upsilon(r)\frac{\partial d}{\partial r} = -\frac{\rho_{gr}}{\rho_p \cdot \varepsilon}\upsilon_{gr}\frac{\partial U}{\partial h} \\ \upsilon_{gr}\frac{\partial\theta}{\partial h} = k_t \cdot (t_c - \theta) \\ \upsilon_{gr}\frac{\partial U}{\partial h} = -k_c(U - b_0 \cdot t^{b_1} \cdot d^{b_2}) \end{cases} . \quad (28)$$

Since the relative speed of the drying agent in the grain layer (filtration rate) varies along the coordinate, it is necessary to take into account the change in the intensity of heat supply to the material and the intensity of moisture removal, which leads to a change in the parameters of the drying agent $t(r)$, $d(r)$. For this, it is necessary to have a dependence of the heat exchange criterion Bi and the mass exchange Bi_m on the filtration speed of the drying agent υ_r, its temperature t for the mass exchange criterion: $\mathrm{Bi}(\upsilon_r)$, $\mathrm{Bi}_m(\upsilon_r, t)$.

The dependence of the Bio heat transfer criterion on the speed of the drying agent (due to the fact that the value of Bi is practically independent of t, d) can be used with the known criterion dependencies Bi(Nu), namely:

$$Bi(Nu) = Nu \cdot \frac{R}{l} \cdot \frac{\lambda_c}{\lambda_g}, \quad (29)$$

where $\mathrm{Nu} = 0.0745 \cdot \mathrm{Re}^{0.625}$; $Re = \frac{\upsilon_r}{\nu}\frac{2}{3}\frac{\varepsilon \cdot d_{e.з}}{1-\varepsilon}$; ε – porosity of the grain layer; $d_{e.g}$ – equivalent diameter of grains; $l = \pi R$; λ_c, λ_g – coefficients of thermal conductivity of the drying agent and grains; ν – coefficient of kinematic viscosity of the drying agent.

From the expanded mathematical expression (29) it follows:

$$\mathrm{Bi}(\upsilon_r) = 0.0745\frac{\lambda_c}{\pi \cdot \lambda_g}\left(\frac{2\varepsilon \cdot d_{e.g}}{3\nu(1-\varepsilon)}\right)^{0.625}\upsilon_r^{0.625}. \quad (30)$$

The dependence $\mathrm{Bi}_m(\upsilon_r, t, d)$ can be determined from the relation for k_c:

$$k_c = \frac{a_m}{(1+Rb^{-1}) \cdot R^2} \cdot \left(0.2 + \frac{1}{Bi_m}\right)^{-1}. \quad (31)$$

Then:

$$Bi_m(\upsilon_r) = \left(\frac{a_m}{(1+Rb^{-1}) \cdot k_c \cdot R^2} - 0.2\right)^{-1}. \quad (32)$$

The drying coefficient k_c is also determined directly from the data of the experimental determination of drying kinetics according to the well-known formula (Kaletnik *et al.*, 2024):

$$k_c = \frac{1}{\tau_c} \cdot ln\frac{u_1 - u_p}{u_2 - u_p}, \quad (33)$$

where τ_c – drying time from grain moisture content u_1 to u_2.

Based on the generalisation of tabular data from the work of P.C. Coradi & Â.F. Lemes (2019), the experimental studies of H. Kaletnik *et al.* (2024), and the approximation of dependence (31), empirical relationships of the drying coefficient with the drying agent parameters were obtained, which, for wheat grain, take the form:

$$k_c(\upsilon_r, t) = 0.025t\upsilon_r^{0.625}, \quad (34)$$

where k_c – drying coefficient (1/h); t – temperature of the drying agent (°C); υ_r – filtration speed of the drying agent, (m/s).

The dependence of the mass transfer criterion Bi_m on the parameters of the drying agent, respectively:

$$\mathrm{Bi_m}(\upsilon_r, t) = \left(\frac{\mathrm{a_m}}{(1+\mathrm{Rb}^{-1})\cdot k_c(\upsilon_r,\mathrm{t})\cdot \mathrm{R}^2} - 0.2\right)^{-1}. \quad (35)$$

The results of solving the system of equations (28) under boundary conditions:

$$r = r_0;\ t(h, r_0) = t_1;\ d(h, R_0) = d_1;\ h = 0;\ \theta(0, r) = \theta_1;\ U(0, r) = U_1.$$

Taking into account (31-32, 34-35), numerical solutions were obtained in the Mathematica computer environment in the form of changes in grain parameters $\theta(r, h)$, $U(r, h)$and drying agent according to the coordinates of the drying space of the bunker dryer. Calculations were carried out with the following ranges of initial data: height of the drying space 6.8 m², thickness of the grain layer $r_0 = 0.4 - 0.6$ m, $R = 1.54$ m specific consumption of the drying agent L = 3,000-4,000 m³/(hour·ton) of dried grain; W_0 – initial moisture 20%. The final grain moisture, which is allowed by the standards for grain dryers 14.0-15.5%, is determined as the grain humidity in the outer grain layer. The temperature of the drying agent at the entrance to the grain material layer was 80-100°C.

A graphical interpretation of the numerical calculation of the wheat grain drying process parameters according to model (28) in the Mathematica mathematical environment is shown in Figure 5. The calculated data correlate well with the experimental data of grain wheat drying in the SBS-5 bunker dryer. The uneven distribution of grain moisture at the dryer outlet is up to 20% ($\Delta\omega = 16.1 - 12.8\%$). The uniformity of drying, i.e. the difference in grain moisture in the zones of the inner and outer cylinder of the dryer, can be increased by increasing the consumption of the drying agent Figure 6, but at the same time the energy losses with the spent drying agent increase several times.

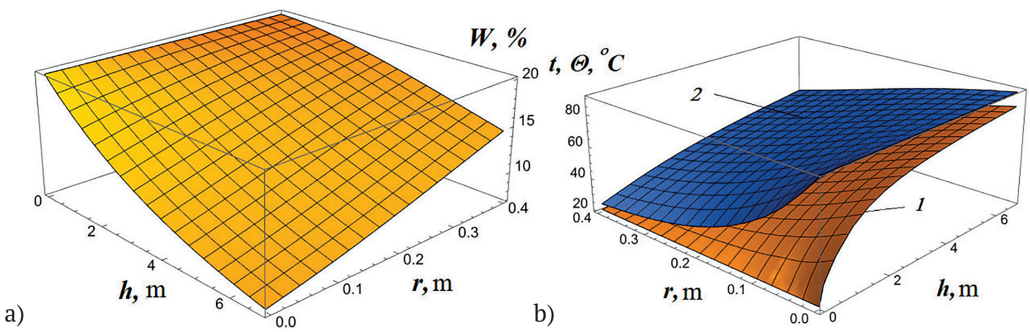

Figure 5. Simulation results with parameters $L = 3{,}000$ m³/(hour·ton); $R - r_0 = 0.4$ m; $W_0 = 20\%$; $t = 90$°C
Note: a) – field of moisture content of the grain layer; b) – temperature fields: 1 – grain layer; 2 – drying agent in the grain layer
Source: compiled by the authors

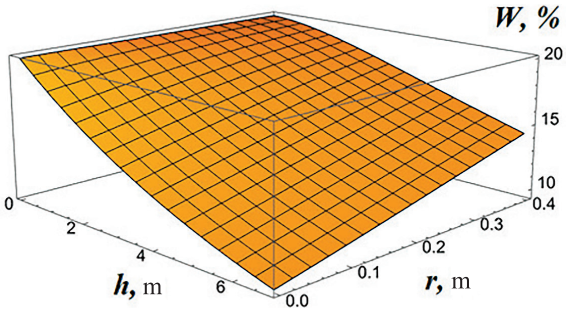

Figure 6. Grain layer moisture content field
Note: $L = 4{,}000$ m³/(hour·ton); $R - r_0 = 0.4$ m $W_0 = 20\%$; $t = 90$°C
Source: compiled by the authors

To simplify numerical calculations of the periodic circulation mode, it was assumed that after each cycle of grain passing through the drying zone, the grain material was completely mixed, and its parameters were averaged along the radial coordinate:

$$U_{out}(r) = \bar{U}_{out.a};\ \theta_{out}(r) = \bar{U}_{out.a}, \quad (36)$$

where $U_{out}(r)$, $\theta_{out}(r)$ – distribution of moisture content and temperature along the radius at the exit from the drying zone when $h = H$. The magnitude of the average values of temperature and moisture content is determined by the obtained formulas:

$$\bar{\theta}_{out.gr} = \frac{2}{R^2}\int_{r_0}^{R} r \cdot \theta(r) dr; \quad (37)$$

$$\bar{U}_{out.gr} = \frac{2}{R^2}\int_{r_0}^{R} r \cdot U(r) dr. \quad (38)$$

The scheme of implementation of the process of periodic-circulating grain drying is shown in Figure 7.

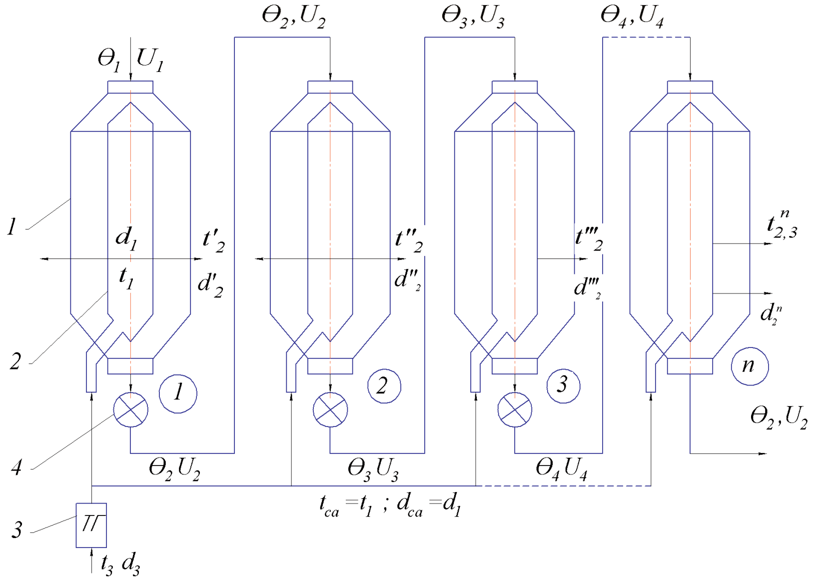

Figure 7. Technological scheme for implementing the process of cyclic drying of grain in a bunker unit
Note: 1 – outer cylinder of the bunker; 2 – inner distribution cylinder; 3 – heat generator; 4 – grain material mixer
Source: compiled by the authors

The schematic diagram of the calculation of the process of periodic-cyclic drying of grain in a moving annular-cylindrical layer is shown in Figure 8, which is based on the mathematical description (model) of the equations of the system (28) taking into account (33), (37), (38).

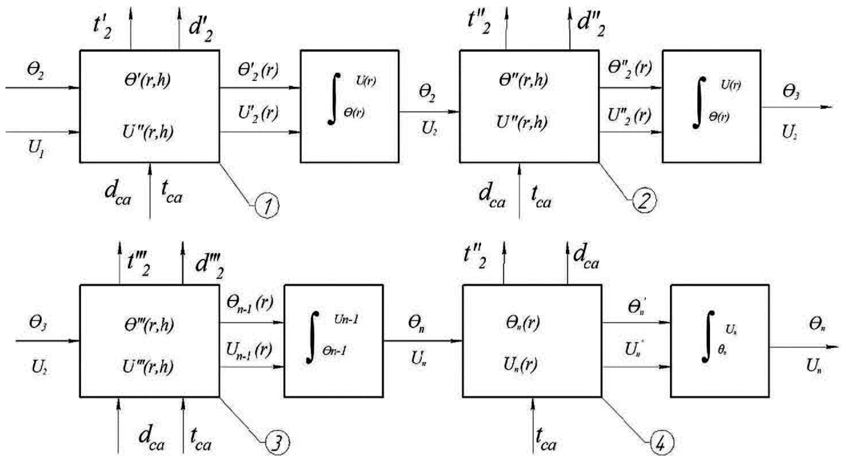

Figure 8. Schematic diagram of the calculation of periodic-cyclic drying of grain in a moving annular layer
Note: 1, 2, 3…n – cycle; θ_k, U_k – final values
Source: compiled by the authors

Calculation algorithm for periodic circulation drying (based on model (28) and illustrated in Figure 9) consists of the following steps:

1. Selection of the number of cycles $n = 1\text{-}5 \dots n(\tau_c)$.
2. Calculation of the speed of movement of a gravitationally directed grain:

$$V_g = \frac{H \cdot n(\tau_c)}{\tau_c}, \qquad (39)$$

where τ_c – drying time; H – height of the drying agent distributor.

3. Calculation in the first cycle of grain drying under the following boundary conditions: $h = 0$; $\theta(r, 0) = \theta_1$; $U(r, 0) = U_1$; $t_{c.a} = t_1$; $d_{c.a} = d_1$; $h = H$; $\theta(r, H) = \theta_2(r)$; $U(r, H) = U_2(r)$.
4. Determination of the average values of θ_2 (r) and $U_2(r)$ of the grain parameters at the exit from the drying zone $h = H$ using equations (37) and (38) $\theta_2(r) = \overline{\theta}_2$ and $U_2(r) = \overline{U}_2$.
5. The obtained average values of $\overline{\theta}_2$ and $2\overline{U}_2$ are required as boundary (initial conditions) parameters of the grain at the entrance to the drying zone during the second drying cycle.

6. Next, the modelling scheme of the process of changing the temperature and moisture content of grain is repeated for the following cycles. The parameters of the drying agent at the entrance to the grain layer remain unchanged throughout the grain drying process.

7. Drying time (residence time) of grain in the dryer after reaching the conditional value of average moisture content – $\bar{U}_k = 17\%$ ($W = 14\%$):

$$\tau = \frac{\sum h \cdot n}{V_g}. \tag{40}$$

Figure 9 shows a test graph of changes in grain parameters along the length of the "drying path" $L = H_c \cdot n$.

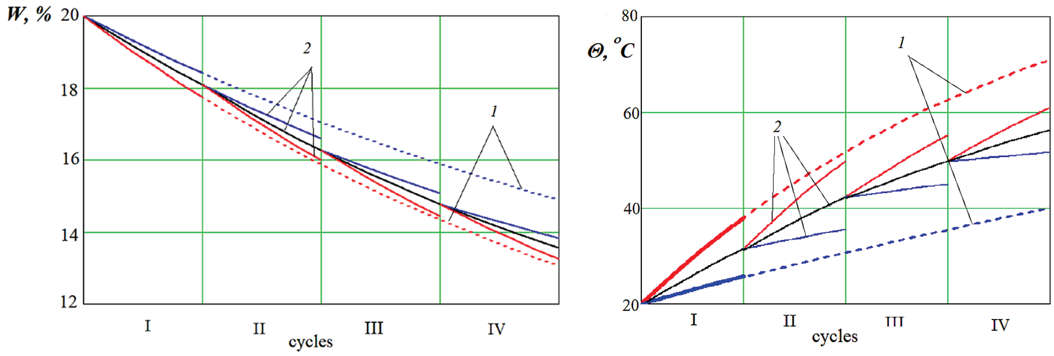

Figure 9. Kinetics of drying (a) and heating (b) of grain

Note: 1 – on the inner and outer wall of the layer in a continuous process, 2 – on the inner, outer and average value in a cyclic-periodic process

Source: compiled by the authors

Convective drying of grain in a dense, thick ring-cylindrical moving layer is a complex physical and technical process accompanied by simultaneous heat and mass transfer and complex dynamic interaction between the solid and gas phases. As substantiated in the study by V. Messerle & A. Ustimenko (2024), an adequate description of this process requires solving a system of nonlinear partial differential equations that take into account external and internal heat and mass transfer, changes in parameters, and hydrodynamic conditions of the medium. Such equations, even in a simplified form, can be effectively solved only by numerical methods. The results of the numerical modelling conducted in this study allowed for a detailed analysis of the patterns of temperature and humidity distribution in a thick grain layer of a bunker dryer with a cross-feed drying agent, and enabled the establishment of optimal drying modes. The proposed model takes into account unsteady processes and features of drying agent filtration in the ring-cylindrical layer, which is an adequate simulation of the geometry of the grain dryer. Preliminary analytical solutions of mathematical models for bodies of classical shape, particularly the work by Y. Sniezhkin & Z. Petrova (2023) provided a theoretical description only for constant thermophysical coefficients. The present work demonstrates the effectiveness of the inverse problem method for identifying thermophysical coefficients based on the experimental kinetics of grain drying and heating. This approach allowed modelling the development of temperature and humidity fields in a single grain modelled as a sphere. However, analytical mathematical solutions are cumbersome and unsuitable for practical engineering use, so it is advisable to use simplified balance models to describe processes in a thick grain layer.

This observation aligns with the findings by Sh.A. Sultanova *et al.* (2024), who demonstrated that energy-based mathematical modelling enables the precise control of temperature fields within a rotary thermosyphon dryer. Their model accounted for the dynamics of heat and mass transfer during the drying of pumpkin seeds, confirming that simulation tools can accurately reflect the kinetics of moisture removal and inform the selection of optimal drying trajectories. Importantly, they highlighted the critical role of boundary conditions – such as internal airflow velocity and rotating drum speed – in determining drying efficiency, suggesting that unified models must be sensitive to geometric and operational variability. A similar perspective is found in the work by H. Kaletnik *et al.* (2024), where a vibratory mill with spatial circulation was applied as the basis for constructing a drying rig. Their research underscored the benefit of incorporating elements of forced particle movement to overcome stagnation zones and non-uniform heat distribution. The proposed technical configuration improved the intensity of convective flows and reduced local temperature differentials, pointing to the advantages of mechanical agitation for energy-efficient moisture extraction. Notably, their design was adapted for granular agricultural products with high bulk density – highlighting the relevance of process-specific parameterisation.

The approach developed in the current study, which recommends precise regulation of bed thickness and filtration velocity, is particularly relevant when integrated with findings from by I. Kuznietsova *et al.* (2020), who applied differential scanning calorimetry (DSC) to evaluate the thermal behaviour of tomato tissue during drying. The DSC analysis enabled the identification of critical temperature thresholds associated with structural collapse or case hardening, indicating that empirical calibration of mathematical models with thermal analysis data can significantly improve predictive capacity. Their results confirmed that even slight deviations in thermal input could result in non-linear responses in moisture migration, which reinforces the need for adaptive control strategies in drying systems. The obtained results confirm the effectiveness of using mathematical modelling to optimise key technological parameters such as temperature, moisture content and duration of drying. A similar modelling approach was applied by R. Chuiuk (2025), who developed a kinetic model of the transesterification process of waste oils into biodiesel. In his work, the dynamic changes in the concentrations of reactants and products were taken into account, along with temperature-dependent reaction rates. This modelling enabled the identification of optimal process conditions to ensure high yield and energy efficiency. These findings further demonstrate the versatility of mathematical models as tools for improving technological processes in different fields.

Taken together, the reviewed studies validate the integration of physical modelling and experimental diagnostics as a robust framework for the development of energy-optimised drying processes. They also emphasise the necessity of tailoring model parameters to reflect material-specific properties and equipment geometry, thereby supporting the direction adopted in this work. Numerical modelling of the effect of drying agent temperature confirmed that increasing the temperature from 80°C to 100°C reduces the drying time by 25-30%, which is consistent with the results of A. Kraiem *et al.* (2023). However, further analysis showed that an excessive increase in the temperature of the drying agent leads to overheating of the grain, especially in areas near the inner cylinder of the dryer (up to 72°C), which can negatively affect the sowing quality of the grain. It is recommended to use a drying agent temperature in the range of 85-90°C, which ensures sufficient process intensity and reduces the risk of thermal damage to the grain. The analysis of the effect of air flow rate confirmed the conclusions of I. Shevchenko & E. Aliiev (2020) that it is inexpedient to increase it above 0.5 m/s, as this leads to an unreasonable increase in energy costs (up to 6 MJ/kg of evaporated moisture) without a significant improvement in dryer performance. The optimum air flow rate was determined to be 0.3 m/s at the entrance to the grain layer up to 0.4 m thick, which ensures an acceptable moisture gradient and minimal energy losses.

Modelling of the drying process for different radial layer thicknesses (0.3, 0.4, 0.5 m) at a drying agent temperature of 90°C and an air flow rate of 0.3 m/s showed that the optimal vertical movement rate of the layer was 0.17, 0.13, and 0.10 m/min, respectively. At the same time, the dryer's productivity was 19.9, 20.2, and 19.5 t/h, the average grain outlet temperature was about 50°C, and the temperature difference between the inner and outer walls increased from 13 to 18°C with increasing layer thickness. The humidity unevenness according to the layer thickness ranged from 6% to 9%, and the energy consumption for moisture evaporation decreased from 5.2 to 4.2 MJ/kg. The results clearly demonstrate the existence of a discrepancy between energy efficiency and drying quality: increasing the layer thickness reduces energy consumption but increases drying unevenness and the risk of overheating of the inner layers, while reducing the thickness leads to lower performance and increased energy losses with the spent drying agent.

The organisation of a cyclic-periodic drying regime with grain stirring is proposed in this paper as a rational solution to eliminate these contradictions. This ensures uniformity of drying with a moisture deviation of 1-1.5% and avoids overheating of the grain material. In addition, stirring promotes the activation of further moisture transfer due to the diffusion of moisture from the inner layers of the grain to its surface during the stirring period, which is confirmed by an increase in the moisture transfer coefficient β by 8-14% after stirring, according to the identified mass transfer criteria Bio for different process regulations.

Comparison of the results with the modes implemented in serial grain dryers by Fratelli Pedrotti, Agrimec, Mecmar, Agrex and others (Fig. 10) showed that although these installations provide high uniformity of drying, the intensity of mixing in them is much higher, which leads to increased grain injury in the augers and increased energy costs for mixing.

a)

b)

Figure 10. Appearance of grain dryers
Note: a) Fratelli Pedrotti XL-420; b) Agrimec AS 1150
Source: Fratelli Pedrotti mobile grain dryers (n.d.), Agrimec grain dryer (mobile) (n.d.)

The results of the study confirm that the specific energy consumption for the evaporation of 1 kg of moisture in the proposed periodic-circulation drying mode ranges from 4.2 to 5.2 MJ/kg, which is fully consistent with the values characteristic of modern energy-efficient grain dryers. This range correlates with the findings of I. Bezbah *et al.* (2022), who developed new dryer constructions aimed at reducing energy costs without compromising drying quality, and with the results of P.C. Coradi & Â.F. Lemes (2019), who applied CFD-based optimisation to improve energy utilisation in silo-dryer systems. The obtained results highlight the high energy-saving potential of bunker-type dryers operating under periodic regimes, especially when combined with adaptive control of the drying agent flow and the vertical movement rate of the grain layer.

An important direction in increasing energy efficiency is the integration of waste heat recovery technologies. The study confirmed the relevance and applicability of such approaches in bunker systems, which resonates with the work by F. Huang *et al.* (2017), who investigated industrial-scale heat recovery schemes, and by Z.Y. Xu *et al.* (2019), who emphasised the feasibility of low-temperature waste heat recovery in agricultural drying systems. According to these studies, up to 20-30% of thermal losses in conventional dryers can be recuperated using plate or rotary-type heat exchangers. In the present research, the modelling results suggest that up to 27% of the exhaust heat energy can be effectively returned to the drying process without compromising the thermal stability of the system. This effect is especially evident when using a two-stage heating zone with variable drying agent temperatures and flow control, which stabilises the internal gradients of temperature and humidity within the grain mass. This effect is especially evident when using a two-stage heating zone with variable drying agent temperatures and flow control, which stabilises the internal gradients of temperature and humidity within the grain mass.

Moreover, the numerical experiments conducted in the Mathematica environment confirm that energy savings are not only a result of improved insulation or material properties, but also strongly depend on the rational configuration of drying cycles and the reduction of peak thermal loads. The integration of heat recovery modules becomes particularly effective when combined with periodic grain mixing, which equalises thermal fields and enhances the efficiency of heat transfer in the layer. These findings support the thesis that future dryer designs should be guided by the principle of thermal self-sufficiency, where the waste heat of the system is partially or fully reintegrated into the process, reducing the reliance on external thermal energy sources. Such an approach is not only economically viable, but also aligns with the broader goals of sustainable agriculture and carbon footprint reduction.

Thus, the numerical modelling confirmed the high sensitivity of the process to the technological and design parameters of the plants and allowed for the formulation of new approaches to the design of efficient grain dryers with radial drying agent feeding. The proposed mathematical models can be used to create automated control systems for the drying process with feedback based on the temperature and humidity of the drying agent and grain, which will help reduce energy costs and ensure consistently high quality of finished products.

CONCLUSIONS

The developed two-level mathematical model of the process of drying seed material in bunker dryers with radial supply of the drying agent allows to adequately describe the kinetics of dehydration and heating of both a single grain and a dense ring-cylindrical layer of grain. This provides a reliable theoretical basis for further optimisation of drying regimes. The thermo physical coefficients of the models are identified based on the results of experimental studies, which allows for quick engineering calculations and prediction of results when changing the input parameters of drying, such as temperature, speed, and moisture content of the agent.

A significant effect of the distribution of temperature and humidity of the drying agent along the radius of the bed was found, which increases the unevenness of drying. The largest energy losses and a decrease in drying quality are observed in a single direct-flow mode due to uneven heating of the grain mass. The use of a periodic-circulating drying mode based on the repeated passage of grain through the drying zone with subsequent mixing is proposed. This allows to significantly reduce the moisture difference between the inner and outer layers of the grain mass and increase the overall energy efficiency of the process. Based on modelling and calculations, the rational design parameters of bin dryers (layer thickness, filtration rate, height of the working area) were determined, which ensure the lowest energy losses while maintaining high grain quality.

Further research should focus on determining the optimal number of grain mixing cycles depending on initial moisture content, considering energy consumption and drying quality. Promising directions include studying heat and mass transfer under variable drying agent feed and grain movement rates, which may enable uniform drying without mixing. This will require refining mathematical models by incorporating dynamic moisture changes into the drying coefficient. Particular attention should be given to the impact of high-temperature regimes on the biological characteristics of grain, including seed viability and nutritional quality. In addition, the synthesis and implementation of automated control systems based on modern microprocessor feedback devices-especially computer vision sensors, real-time grain moisture sensors, and artificial intelligence elements – remain highly relevant for adaptive regulation of drying parameters.

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CONFLICT OF INTEREST

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Математичне моделювання та дослідження процесу сушіння зерна в бункерних установках з радіальною подачею сушильного агента

Анотація. Одним із ключових завдань сучасного зернопереробного обладнання є підвищення ефективності сушіння в бункерних установках із кільцево-циліндричним шаром зерна. Метою даної роботи було дослідження нестаціонарних процесів тепломасообміну в кільцевому шарі зерна в сушарці бункерного типу з радіальною подачею сушильного агента для оцінки рівномірності розподілу температури і вологи та визначення впливу параметрів сушіння на ефективність зневоднення. Для підвищення енергоефективності та якості сушіння запропоновано дворівневу математичну модель, яка дозволяє дослідити процеси внутрішнього тепломасопереносу та зовнішнього тепло- і масообміну. На першому рівні моделювалася кінетика сушіння окремої зернівки у вигляді сфери, що контактує з сушильним агентом постійної температури й вологості. На другому рівні описано процес фільтраційного сушіння щільного кільцево-циліндричного шару зерна при радіальній подачі сушильного агента. Теплофізичні коефіцієнти моделей ідентифіковані за результатами натурних експериментів. Отримані аналітичні та чисельні розв'язки представлених математичних моделей дозволили визначити раціональні параметри конструкції та розраховувати енергоефективні режими роботи зерносушарок бункерного типу в залежності від початкових параметрів зерна. Обґрунтовано, що найбільша ефективність досягається при використанні періодично-циркуляційного режиму, що забезпечує рівномірне прогрівання та зневоднення зернової маси за радіальною координатою шару. Встановлено залежності впливу температури, вологовмісту та швидкості фільтрації повітря на експозицію сушіння зерна в шарі, що дозволяє здійснювати гнучке керування сушильним процесом залежно від умов експлуатації. Результати дослідження підтвердили, що застосування періодично-циркуляційного режиму сушіння дозволяє досягти високої рівномірності кінцевої вологості зерна, підвищити енергоефективність процесу та значно знизити нерівномірність нагріву зерна в порівнянні з традиційними безперервними режимами сушіння зерна в товстому радіальному шарі. Результати можуть бути використані як для модернізації існуючих сушильних установок, так і при проектуванні нових енергоощадних зерносушарок

Ключові слова: математичне моделювання; тепломасообмін; бункерна сушарка; циклічно-періодичний режим; кільцево-циліндричний шар